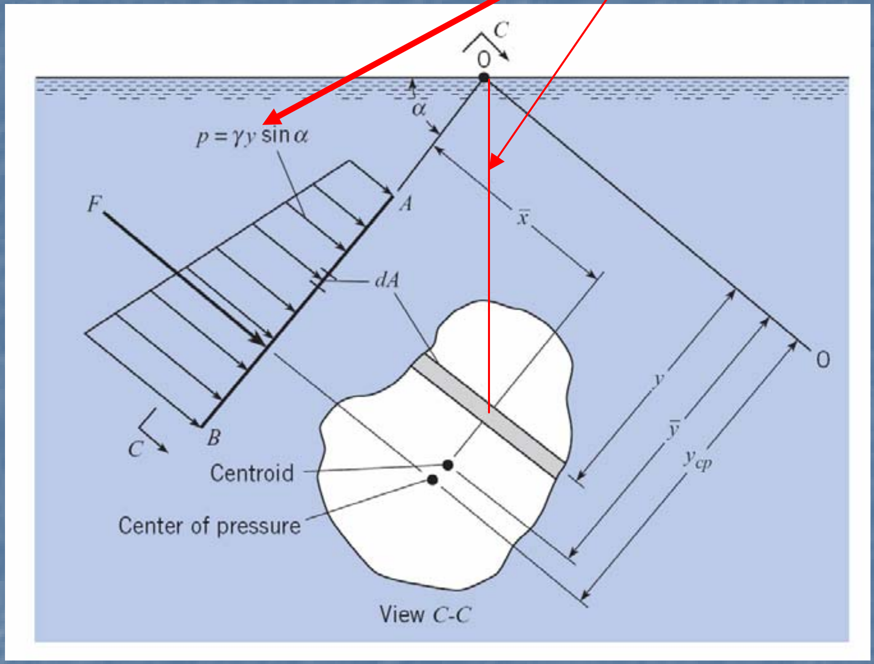
Fluid Statics

PRESSURE FORCES ON PLANE SURFACES

$h = y \sin\alpha$

The resultant hydrostatic force



$$F = \int_A p\,dA = \int_A \gamma y \sin\alpha\, dA = \gamma \sin\alpha \int_A y\,dA = \gamma\, \bar{y} \sin\alpha A = \bar{p}\, A$$

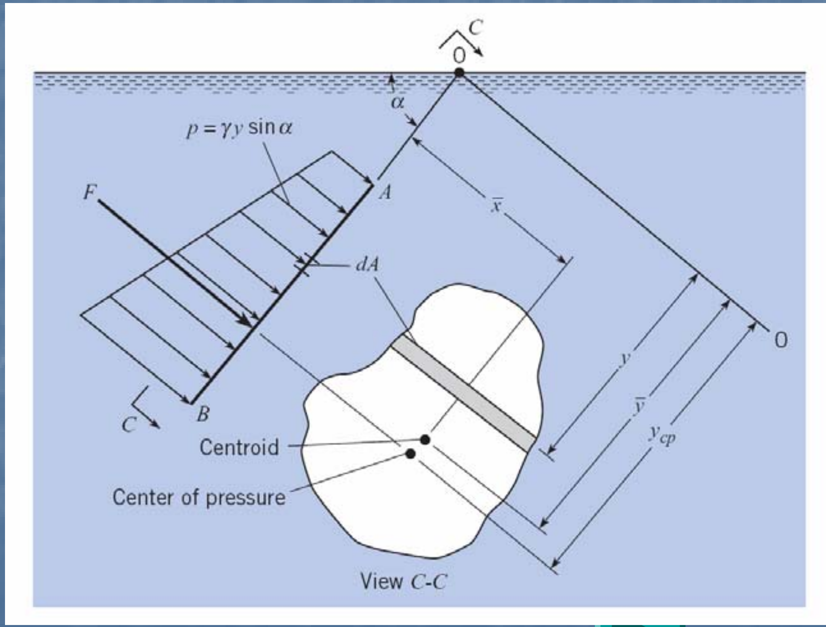
Where $\bar{p}$ = pressure at the center of gravity of the surface.

Always Re*meber* $\bar{y}\sin\alpha = h_{CG}$ = *Vertical dis*tan*ce fromthe surfasce*

Fluid Statics

LINE OF ACTION OF RESULTANT VERTICAL HYDROSTATIC FORCE ON PLANE SURFACES

$$y_{cp}F = \int y\,dF = \int y\,(p\,dA) = \int y(\gamma y \sin\alpha)\,dA = \gamma \sin\alpha \int y^2\,dA$$



Where ($\int y^2 dA$) is the second moment of area = $I_0 = \bar{I} + \bar{y}^2 A$

$$y_{cp}F = \gamma \sin\alpha(\bar{I} + \bar{y}^2 A)$$

$$y_{cP}(\gamma\,\bar{y}\sin\alpha)A = \gamma \sin\alpha\left(\bar{I} + \bar{y}^2 A\right)$$

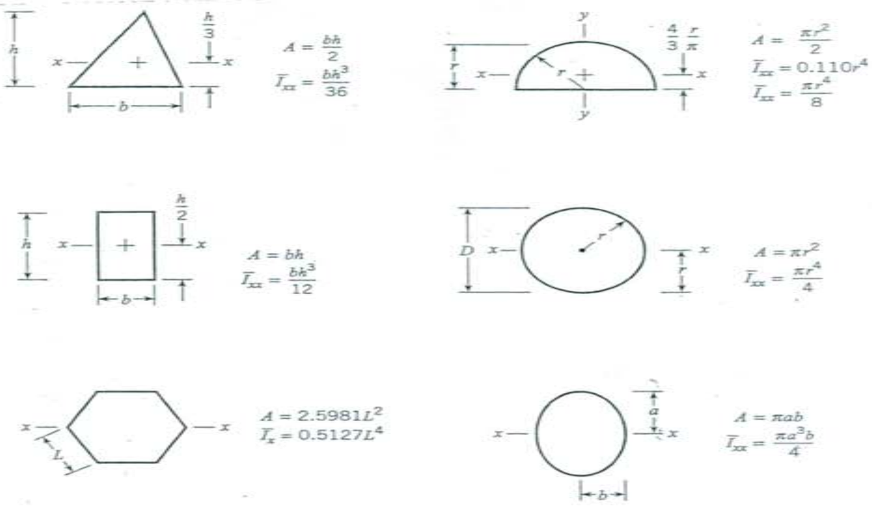
$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A}$$

Figure (A5)

Centriods and moments of areas of plane areas

A.1

s and moments of f plane areas



Volume and Area Formulas:

$A_{circle} = \pi r^2 = \pi D^2/4$

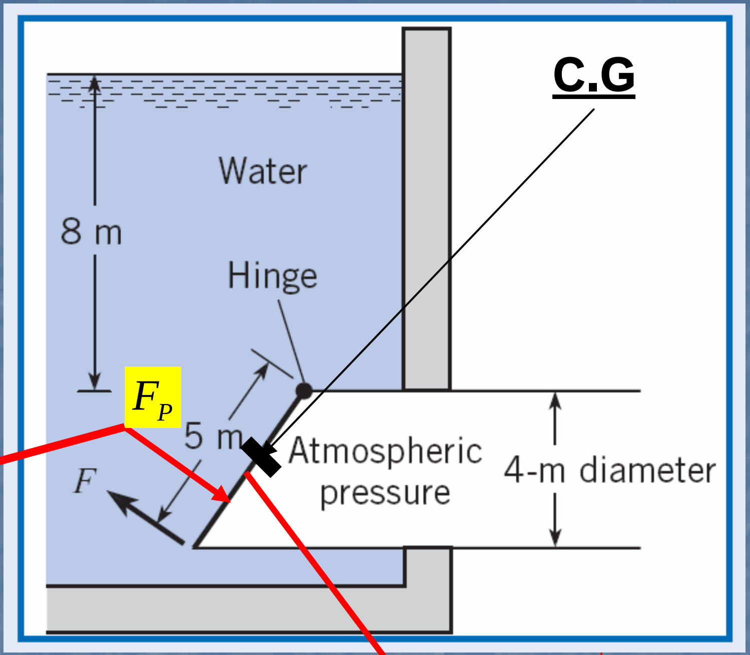
$A_{sphere\ surface} = \pi D^2$

$V_{sphere} = \frac{1}{6}\pi D^3 = \frac{4}{3}\pi r^3$

Example(3.12)

Calculate (F) to open the gate

Neglect weight of the gate

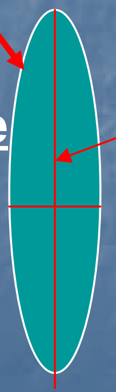


Hydrostatic Force due to pressure =

$$F_p = \bar{p}A = \gamma(h_{CG})A = \gamma(\bar{y}\sin\alpha)A$$

$$\sin\alpha = (4/5)$$

Elliptical gate



D = 4m

Always Remeber $\bar{y}\sin\alpha = h_{CG}$ = *Vertical distance fromthe surfasce*

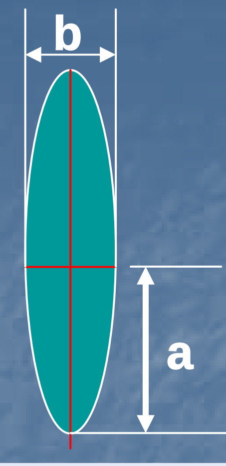
Solution First evaluate the magnitude of the hydrostatic force:

$$F = \bar{p}A$$

The area in question is an ellipse with major and minor axes of 5 m and 4 m. The area i given by the formula $A = \pi ab$ (from Fig. A.1 in the Appendix). Then

$$F = 10 \text{ m} \times 9810 \text{ N/m}^3 \times \pi \times 2 \text{ m} \times 2.5 \text{ m} = 1.541 \text{ MN}$$

$$F_p = \bar{p}A = \gamma(h_{CG})A = \gamma(\bar{y}\sin\alpha)A$$



Now calculate the slant distance between the centroid of the elliptical area and the center of pressure:

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{\frac{1}{4}\pi a^3 b}{\bar{y}\,\pi ab} = \frac{\frac{1}{4}a^2}{\bar{y}}$$

Here $\bar{y} = 12.5$ m (slant distance from the water surface to the centroid). Thus

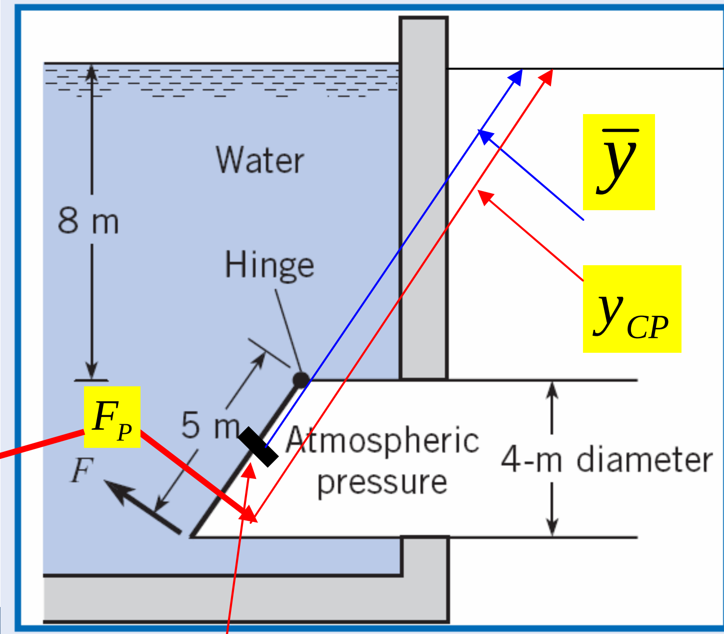
$$y_{cp} - \bar{y} = \frac{1}{4} \times \frac{6.25 \text{ m}^2}{12.5 \text{ m}} = 0.125 \text{ m}$$

Now take moments about the hinge at the top of the gate to obtain F:

$$\sum M_{\text{hinge}} = 0$$

$$1.541 \times 10^6 \text{ N} \times 2.625 \text{ m} - F \times 5 \text{ m} = 0$$

$$F = 809 \text{ kN}$$



C.G

Example(3.12)

Determine the magnitude of the hydrostatic force acting on one side of the submerged vertical plate shown in the figure and determine the location of the center of pressure.

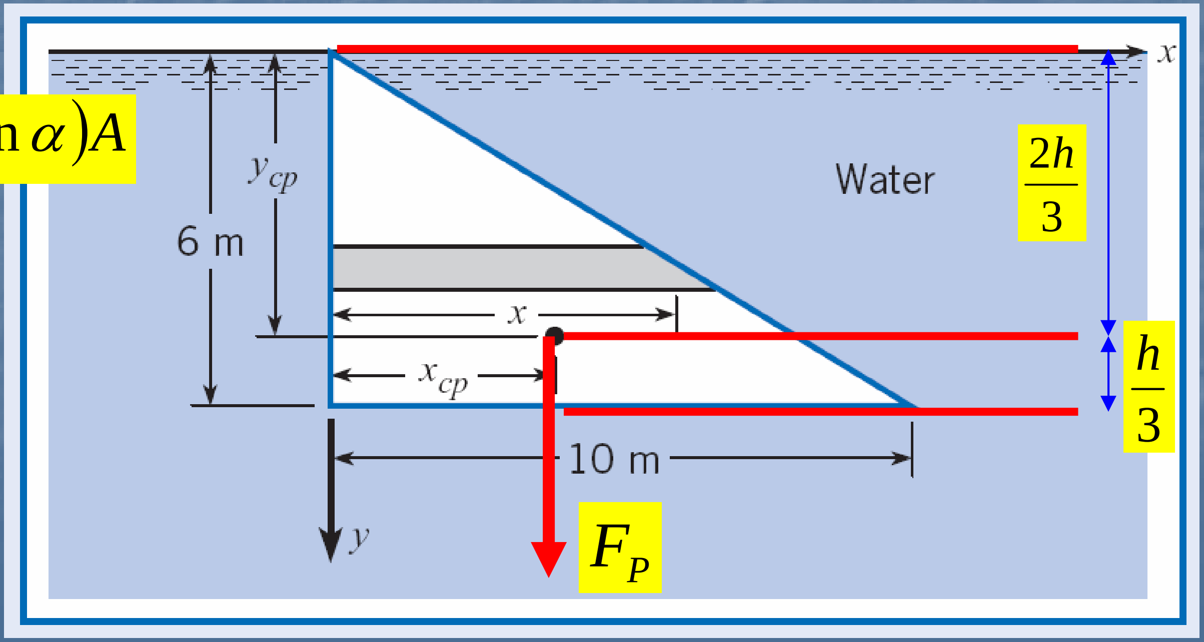
Solution The centroid of the plate is at a depth of 4 m. Therefore $F = 4\ \text{m} \times 9810\ \text{N/m}^3 \times \frac{1}{2} \times 60\ \text{m}^2 = 1.177\ \text{MN}$. The vertical location of the center of pressure is obtained from the center-of-pressure equation:

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{bh^3/36}{\bar{y}\frac{1}{2}bh} = \frac{h^2}{18\bar{y}} = \frac{36}{72}$$

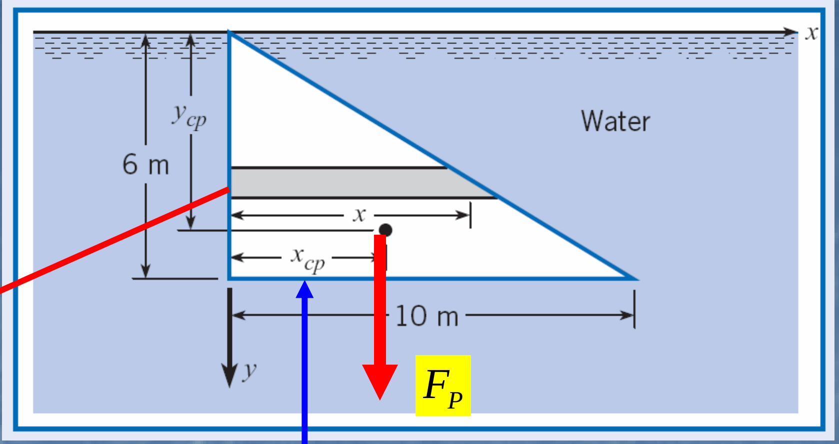
$$y_{cp} = 4 + \tfrac{1}{2} = 4.50\ \text{m}$$

$$F_p = \bar{p}A = \gamma(h_{CG})A = \gamma(\bar{y}\sin\alpha)A$$

$$Note: \alpha = 90^\circ$$



Vertical edge



Obtain the lateral location of the center of pressure by summing moments of forces ing on the elemental strips and then dividing by F. Moments are taken about the ve edge:

$$dM = \tfrac{1}{2} x\, dF = \tfrac{1}{2} x \gamma y x\, dy$$

$\bar{p}$ A

But
$$x = \frac{10}{6} y$$

so
$$M = \frac{50}{36} \gamma \int_0^6 y^3\, dy$$

Then
$$M = \frac{50}{36} (9810\ \mathrm{N/m^3}) \frac{y^4}{4} \Big|_0^6 = 4.414\ \mathrm{MN \cdot m}$$

But
$$F x_{cp} = M$$

so
$$x_{cp} = \frac{M}{F} = \frac{4.414\ \mathrm{N \cdot m}}{1.177\ \mathrm{N}} = 3.75\ \mathrm{m}$$

END OF LECTURE

(4)